DECENTRALIZED SPACING CONTROL WITH COMMUNICATION DELAY: A STATE SPACE MODELING BASED DESIGN APPROACH

Thesis submitted in partial fulfilment of the requirements for the degree of

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In

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(Control & Automation)

By

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Certificate

This is to certify that the work in this thesis entitled **Decentralized Spacing Control with Communication Delay: A State Space Modeling Based Design Approach** by **Aditya Kumar** is a record of an original research work carried out by him during 2013-2014 under my supervision and guidance in partial fulfilment of the requirement for the award of the degree of Master of Technology in Electrical Engineering (Control & Automation), National Institute of Technology, Rourkela. Neither this thesis nor any part of it, to the best of my knowledge, has been submitted for any degree or diploma elsewhere.

Place: NIT Rourkela

Date: -----May 2014

Prof. Sandip Ghosh

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ABSTRACT

Spacing control is important topics of research as increasing number of vehicle are automatically controlled in land, space and under water. Spacing control has importance as it helps in safety of passengers and also saves time and fuel. It has application in many fields like aircraft flight formation vehicle platooning, spacecraft, etc. A platoon is group of vehicles travelling with a leader which is followed by the others. Information of the leader is very important in case of platoon to be safe. Vehicles are equipped with wireless communication which is used in sending and receiving information from the other vehicle and specially leader information. These communication systems are not always prefect, sometime or frequently it gets affected by environmental factors and is not able to send and receive information. The need of robust control law to minimize the distance between vehicles and that remains robust in uncertain conditions is important.

In this research Switched Static Output Feedback Stabilization with LMI approach has been used. This method has advantages over earlier method that has limitation over switching sequence. Using SOF method gives us a simple control methodology that doesn't need state information and LMI method is reliable in finding stability condition. It results in a control method that will stabilize the system in any arbitrary switching. System will attain string stability as it will ensure that error between vehicles doesn't increases as we go down the platoon [1].

Contents

SYMBOLS AND ABBREVIATIONS

s_i	Position of vehicle i
v_i	Velocity of vehicle i
a_i	Acceleration of vehicle i
δs_i	Spacing Error of vehicle i
l_i	Desired Spacing between vehicle i and vehicle $i-1$
$\|\delta s_{i-1}\|_\infty$	Infinity Norm of Signal δs_i
$\|h(k)\|_1$	1^{st} Norm of impulse response $h(k)$
$\hat{v}$	Estimation of v
$\bar{v}$	Propagation of v
$\tilde{v}$	Error $\hat{v} - v$
$\breve{v}$	Measurement of v
ϵ	Belongs to
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	The set of real n vectors
$\mathbb{R}^{n\times m}$	The set of real $n \times m$ vectors
LMI	Linear Matrix Inequality
BMI	Bilinear Matrix Inequality
SOF	Static Output Feedback Stability

CHAPTER 1

INTRODUCTION

1.1 Background

Platoon of vehicle is multiple automated vehicle travelling in a desired pattern, maintaining space between vehicles have been important problem from long time and extensive research are carried out. Earlier research has taken neighbour as reference, in this approach vehicle follows vehicle just ahead of it. But in this case, if preceding vehicle has an initial error following vehicle may get more error and it may go on increasing as we go down the platoon. In 1960’s shladover [2] pointed out the drawback of headway control. Lead vehicle reference has been given, where every vehicle in platoon follows the lead vehicle and preceding vehicle. String stability concept was developed so that error of vehicle should go on decreasing as we go down the platoon. It requires information (position, speed, acceleration) of lead vehicle as well as preceding vehicle. Vehicle in platoon communicates through wireless data links. String stability is not possible in case of unavailability of lead vehicle information which leads to study of string stability with communication delay.

Effects of communication delay have been studied by Xlangheng Liu and Andrea Goldsmith [3]. They has given a detailed information of string stability of platoon when complete information of lead vehicle is available. He also pointed out the behaviour of platoon when there is communication delay, stability at the time of communication is delay is difficult as system vehicles have to estimate the information and that makes it hybrid system.

Later in 2009 Rodney Teo and Claire J Tomlin [4] designed Control law for system with lead vehicle information dropout. They have taken assumptions about pattern of dropouts in lead vehicle information. Teo and Tomlin has given a control law that string stabilizes the system and provided detailed information about worst case condition.

1.2 Decentralized Control

While dealing with control problems three steps: modeling, describing qualitative properties and controlling system behaviors are applied. This concept is applicable for centralized control, where a single controller is designed based on whole system information. But centralized control is not reliable and economical for the implementation into large scale system and also increases complexity in the design process. Because there is possibility of losing local data, presence of time delays due to long distance information transfer and presence of uncertainty in the model. Thus, the control problem becomes too large to be controlled and too complex to be solved. Whereas decentralized approach [5, 6] provides a way to deal with above difficulties by breaking the original system into a no. of subsystem. Each subsystem is controlled by a local controller, which requires a part of global information. Thus decentralized control design solves difficulties encountered in analyzing, designing and implementing control strategies and algorithms in centralized case.

1.3 Spacing Control

Spacing control is maintaining desired space between two vehicles in a platoon. Spacing control is being studied to find control laws that can minimize the space between two vehicles to increase the highway capacity as no of vehicles are increasing rapidly. In order to maintain gap between vehicles we need to transmit lead vehicle and preceding vehicle information. Earlier research had different approaches.

- ❖ Unit center referenced: - In this system vehicle in center is taken as reference in order follow the defined formation.
- ❖ Neighbored referenced: - every vehicle has its Neighbor as reference and it only receives information about these vehicle.

- Leader referenced: - It has leader vehicle that is in front of all vehicle and every vehicles follows and receives information about it and also about the neighbor which makes it more suitable and reliable. Importance of leader information has been studied in many researches.

In this research we will be considering leader referenced as it has advantages over other approach as string stability can't be achieved without lead vehicle information.

1.4 Static Output Feedback Stabilization

The static output feedback control is an important and difficult problem at present time, but it is simple in use that is the reason of its importance. It is less expensive and simple to use as compared to State Feedback Control. In state feedback control states of system are not always available and need expensive state observer that also complicates the system whereas in SOF output is always available. But finding static output controller is not easy and maximum cases it ends in creating a new SOF problem. There have been many dedicated research about SOF but it has not been able to find a universal solution.

The necessary and sufficient condition for the stability of static output feedback controller has been given in [7], for discrete time system it has been studied in [8].

Problem of finding SOF can be simply modified as problem of Bilinear Matrix inequality [BMI]. But solving BMI [9] [10] for convex optimization is not simple but there are methods to convert these BMI problem in to LMI problem which is can be easily optimized to find the solution. In our condition we have discrete time switched system and we need LMI condition for static output feedback controller that has been given in [11]. Convex optimization can be applied in these conditions.

1.5 Convex Optimization

Definition 1.1: A set C convex is convex if the line segment between any two points in C lies in C and following condition satisfies

$$\gamma x_1 + (1-\gamma)x_2 \in C$$

For anyx_1, $x_2 \in C$ and $\gamma$with $0 \leq \gamma \leq 1$.

Definition 1.2: A function $f: R^n \rightarrow R$ is convex if $dom\ f$ is convex set and the following holds:-

$$f(\gamma x_1 + (1-\gamma)x_2) \leq \gamma f x_1 + (1-\gamma)x_2)$$

For anyx_1, $x_2\ \in dom\ f$ and $\gamma$with $0 \leq \gamma \leq 1$.

We can say geometrically that the line segment between $(x, f(x))$ and $(y, f(y))$ lies above the graph of f. fIs a concave function it is replaced by $-f$an affine system holds the above inequality, so all affine system is both convex and concave.

Definition 1.3: - optimization problem a convex function $f: R^n \rightarrow R$ to be minimized over optimization variable x subject to inequality constraint on affine function of x is a convex optimization problem. i.e.

$$minimize\ f(x)$$

Subject to $g_i(x) \leq 0, (i = 0,1,2 \ldots \ldots \ldots \ldots \ldots m)$

$$h_j(x) = 0, (j = 0, 1, \ldots \ldots \ldots p)$$

Where equality constraints are replaced by pair of inequality constraints $h_j \leq 0$and$h_j \geq 0$.

1.6 Motivation

As spacing control has wide uses and automated vehicle are becoming a new trend. Extensive research are carried out on its different aspects. One important problem is about communication through wireless data links suffers from losses which makes delay in lead vehicles information. Research done by Rodney Teo and Claire J Tomlin [4] gave concept of stability of system with communication delay. As system becomes a hybrid system due to the presence of both continuous and discrete signals, one may need we need a switching sequence representation of system. However in hybrid system switching sequence is not known a priory. In [4] they have assumed that switching has a pattern but that is not real. It has been assumed that time difference between two consecutive communication dropouts is equal to or more than the settling time of the system without dropout. But this may not occur as expected.

Here we have tried to find out control law that stabilize the system irrespective of arbitrary sequence. As LMI is a convex approach to find a solution and we apply some earlier research for the stabilization of discrete time switched linear system using LMI approach, which is applicable to spacing control problem.

1.7 Outline of Thesis

- Chapter 1: It contains brief information about the Research Background, Decentralized Control, Spacing Control and Convex Optimization.
- Chapter 2: It focuses on Kinematics and Dynamics of Platoon Formation.
- Chapter 3: It contains concept of String Stability, Effect of Communication Delay on String Stability and earlier method of control using Steepest descent.
- Chapter 4: This chapter contains State Space Approach of Stability using LMI. Theories to find LMI condition of stability.

- Chapter 5: This chapter has numerical analysis, Simulation results, Conclusion and Future Work.

CHAPTER 2

PLATOONING SYSTEM AND EFFECT OF COMMUNICATION DELAY

2. 1 Platooning System Dynamics

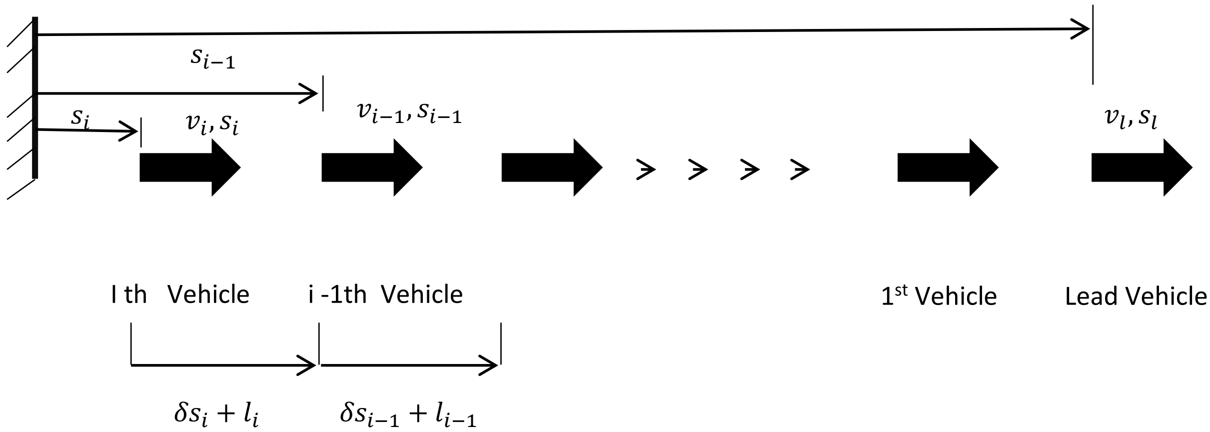

Figure 1 Platooning system

Platooning system is flock of multiple vehicles travelling in a desired pattern. A platoon always has a predefined spacing between the vehicles. In figure 1 we have N no of vehicle travelling in a straight line formation and formation method is lead referenced method. Every vehicle will be following lead vehicle and will also maintain distance with preceding vehicle taking reference of lead vehicle. All vehicles receive information of lead vehicle's (distance, speed, acceleration) and preceding vehicle information.

In the given platoon system of fig.1 we have N no of vehicle following the lead vehicle with maintaining predefined distance between neighbouring vehicle. Where l_i . Is desired spacing between two consecutive vehicle i and $i-1^{th}$ vehicle.

With,

v_i– Velocity of i^{th} vehicle.

s_i- Position of i^{th} vehicle where

$i \in \{l, 1,2,3 \ldots\ldots\}$ (Where$l$refers to lead vehicle).

State of the system can be written as

$\dot{s}_i = v_i$ 2.1

$\dot{\boldsymbol{v}}_i = \boldsymbol{a}_i = \boldsymbol{u}_i$ 2.2

Where $\boldsymbol{a}_i$ is acceleration of thevehicle i . It is also the control input of vehicle as speed of any vehicle can only be controlled by the applied thrust.

Now make an assumption that system is discrete time system with sampling time T and the control input of vehicle i's at time k, $u_{i,k}$,. Then we can write discrete time dynamics of vehicle i as follows.

$\boldsymbol{a}_{i,k} = \boldsymbol{u}_{i,k}$ 2.3

1. $v_{i,k+1} = v_{i,k} + a_{i,k}T$
2. $s_{i,k+1} = s_{i,k} + v_{i,k}T + a_{i,k}\,{T^2}/{2}$ 2.4

Let the relative states be

1. $\boldsymbol{\delta v_i = v_{i-1} - v_i}$ 2.5
2. $\boldsymbol{\delta s_i = s_{i-1} - s_i - l_i}$ 2.6

On substituting of states in equation (2.1) with relative states in discrete time dynamics of system gives relative state dynamics.

$$\boldsymbol{\delta s_{i,k+1} = \delta s_{i,k} + \delta v_{i,k} T} + \tfrac{\boldsymbol{t^2}}{\boldsymbol{2}}(\boldsymbol{u_{i-1.k} - u_{i,k}}) \quad 2.7$$

2.1.1 Control Input

In order to maintain the platoon we need lead and preceding vehicles information. Control input will depend on distance, speed, acceleration of all three vehicle lead, preceding and vehicle$i$. Control input is directly proportional to the distance, speed, acceleration of all three vehicles. It is linear combination of all. Then the control input $u_{i,k}$[12] [13] [14]. Would be

$$\boldsymbol{u_{i,k} = k_1 a_{l,k} + k_2(v_{l,k} - v_{i,k}) + k_3\left(s_{l,k} - s_{i,k} - \textstyle\sum_{j=1}^{i} l_j\right) + k_a a_{i-1,k} + k_v \delta v_{i,k} + k_p \delta s_{i,k}}$$

2.8

To the system we need to substitute the control input from equation(2.5) in relative state dynamics equation(2.4) that will give us the transfer function between δs_{i-1} and δs_i. That is transfer function of vehicle$i$ to the error of vehicle i-1.

For the first vehicle behind the lead vehicle, $i = 1$.

$$\boldsymbol{\delta s_{1,k+1} = 2\delta s_{1,k} - (k_v + k_2)T\delta s_{1,k} - \frac{T^2}{2}\left(k_p + k_3\right)\delta s_{1,k} - \delta s_{1,k-1} + (k_v + k_2)T\delta s_{1,k-1} + \frac{T^2}{2}\left(k_p + k_3\right)\delta s_{1,k-1} + \frac{T^2}{2}(1 - k_a - k_1)a_{l,k} + \frac{T^2}{2}(1 - k_a - k_1)a_{l,k-1}} \quad \mathbf{2.9}$$

2.1.2 Discrete Time Transfer Function

Solving the equation (2.9) and taking z transform we get the transfer function of the first vehicle with respect to acceleration of lead vehicle.

$$\boldsymbol{\frac{\delta s_1(z)}{a_l(z)} = \frac{n^l(z+1)}{z^2+d_1 z+d_0}} \quad \mathbf{2.10}$$

To find a general transfer function for any vehicle $i = (0, 1, 2 \ldots \ldots \ldots i)$ writing equation (2.9) for vehicle $i - 1$ and combing we get

$$\boldsymbol{\frac{\delta s_i(z)}{\delta s_{i-1}(z)} = \frac{n_2 z^2+n_1 z+n_0}{z^2+d_1 z+d_0}} \quad \mathbf{2.11}$$

Where,

$n_2 = k_a$, $n_1 = -2k_a + k_v T + \frac{1}{2}k_p T^2$, $n_0 = k_a - k_v T + \frac{1}{2}k_p T^2$, $n^l = \frac{T^2}{2}(1 - k_a - k_1)$.

$d_1 = -2 + (k_v + k_2)T + \frac{1}{2}T^2(k_p + k_3)$,

$$d_0 = 1 - (k_v + k_2)T + \frac{1}{2}T^2(k_p + k_3)$$

2.2 String Stability

String stability is an important criteria that guarantees the controlled space between vehicles. This concept has many advantages over headway control that needed only preceding vehicle information. For system to be string stable we need lead vehicle information. The concept of string stability is well known and has many uses at present time. It is used for stability in flocking of vehicle but it has other uses like flight formation, in spacecraft. But basic use in platoon of vehicle. Error of vehicle should go on decreasing as it propagates down the platoon. String stability guarantees the stability throughout the platoon and every subsystem as it decreases the peak spacing error as we go down the platoon [16].

In this particular case, peak spacing error between lead vehicle and vehicle 1 δs_1 should always be greater than the peak spacing error between vehicle 1 and vehicle 2 δs_2 . In general we can say that peak spacing error between vehicle $i-1$ and $i-2$ should be less than that between vehicle i and $i-1$ i.e.

$$\|\boldsymbol{\delta s_i}\|_\infty \leq \|\boldsymbol{h}(\boldsymbol{k})\|_{\boldsymbol{1}} \|\boldsymbol{\delta s_{i-1}}\|_\infty \qquad \textbf{2.12}$$

In another words gain of δs_i to δs_{i-1} that is gain of transfer function h (z) in (2.6) should be less than 1.

$$\boldsymbol{h}(\boldsymbol{z}) = \frac{\boldsymbol{\delta s_i(z)}}{\boldsymbol{\delta s_{i-1}(z)}} \leq \boldsymbol{1} \qquad \textbf{2.13}$$

2.2.1 String Stability without Delay

First let's discuss about string stability of vehicle without communication delay that will give us an idea to compare both cases more efficiently. As a delay always degrades performance of our system we need a condition that will insure string stability in these cases. Let's assume condition when we have information of lead and preceding vehicle's information (position, velocity and acceleration), then system will be string stable if only if error decreases as we go down the platoon. For example if we are taking any vehicle that is vehicle $i$in the weak sense it is string stable if ($\|h(k)\|_1 \leq 1$ for any parameter set). After we take lead vehicle information it is stable for appropriate parameters

2.2.2 Effect of communication delay on String stability

As we know real time communication of vehicle is always through the wireless data links. That is not as reliable as wired communication and lacks in some sense. Wireless networks lags or losses data packet as it is in discrete form. And that is not predictable easily in maximum cases. Wireless communication depends on many factors like power fluctuation, scattering, antennas. This in turn limits its ability to continuously transmit signal without any loss or of packet of data. Delay is also arbitrary and can't be determined before, it need to be compensated in terms of continuous operation. In our case string stability depends on control input and it is linear combination of lead and preceding vehicles information, if due to data packet losses we are not able to make the system string stable. Communication delay has very high impact over string stability. Here we have different condition of delay

1) When only lead vehicle information has losses of data packets.
2) When both lead vehicle and preceding vehicle information has suffered packet losses.

When both losses data it is difficult to make system string stable for that we would be taking only one condition in consideration.

$$\frac{\delta s_i(z)}{\delta s_{i-1}(z)} = \frac{n_2 z^2 + n_1 z + n_0}{z^2 + d_1 z + d_0} \qquad \mathbf{2.14}$$

Gain of system should be less than 1.

$$\|\boldsymbol{h}(\boldsymbol{k})\|_1 \leq \mathbf{1} \qquad \mathbf{2.15}$$

With appropriate values ofn_1,n_2,n_0,$d_1$and d_0 can guarantee the stability of platoon. That says error would get decaying down the platoon or in other sense it won't increase along the platoon.

2.2.3 System with Communication Delay

Here we will discuss about string stability when there is packet losses in communication. Preceding vehicle information is taken as less noisy or less delay due to neighbour and distance is measured using infrared. Here we are only considering lead vehicle communication delay.

In case of dropouts we need to estimate the information of lead vehicle's information. We need to design control law with these estimate that will ensure the stability. We can take propagated information and previous information to roughly estimate value at current instant.

$$\hat{v}^i_{l,k} = j^i_k \bar{v}^i_{l,k} + \left(1 - j^i_k\right) \breve{v}_{l,k} \qquad 2.16$$

$$\hat{s}^i_{l,k} = j^i_k \bar{s}^i_{l,k} + \left(1 - j^i_k\right) \breve{s}_{l,k} \qquad 2.17$$

Where $j^i_k = 1$is the lost link between lead and ith vehicle, $j^i_k = 0$implies a good link and doesn't have any losses. $\hat{v}$ stand for the estimate of v , $\bar{v}$ to a propagation of v , ,$\tilde{v}$ is error in the estimate $(\hat{v} - v)$ and $\breve{v}$ to a measurement value of v. Propagation can also be written as the following

$$\bar{v}^i_{l,k+1} = \hat{v}^i_{l,k} \qquad 2.18$$

$$\bar{s}^i_{l,k+1} = \hat{s}^i_{l,k} + \hat{v}^i_{l,k} T \qquad 2.19$$

Now we will use estimation in the transfer function instead of those values to get the transfer function when it losses data. On Substitution of estimates equation (2.18) and (2.19) in control input of system (2.8).

On applying estimation for lead vehicle information we get different control input and our system becomes a hybrid system as it has continuous time dynamics as well as discrete time dynamics. System becomes discrete time switched linear system.

2.3 Switched Linear System

After applying estimation of lead vehicle information system has two dynamics, one when system receives complete information of the lead vehicle and another when information are estimated. System is hybrid system and this class of hybrid system is called discrete time switched linear system. Here we have dynamics of discrete time switched linear system.

$$\boldsymbol{x_{k+1}} = \boldsymbol{A_{\theta(k)} x(k)} + [\boldsymbol{b_{\theta(k),\delta s_{i-1}}}, \boldsymbol{b_{\theta(k),a_{l,}}}]\boldsymbol{u_k} \quad \textbf{2. 20}$$

$$\boldsymbol{y(k)} = \boldsymbol{c_{\theta(k)} x(k)} \quad \textbf{2. 21}$$

Where

$$A_{\theta(k)} = \begin{bmatrix} -d_1 & -d_0 & n_1 & n_0 & \theta_{1,k} & \theta_{0,k} & \varphi_{1,k} & \varphi_{0,k} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j_k^{i-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j_k^i & 0 & 0 \\ 0 & 0 & 0 & 0 & Tj_k^{i-1} & 0 & j_k^{i-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & Tj_k^i & 0 & j_k^i \end{bmatrix}$$

$$u_k = \ [\delta s_{i-1,k-1}, a_{l,k-1}]^T$$

$$x_k = [\delta s_{i,k}, \delta s_{i,k-1}, \delta s_{i-1.k}, \delta s_{i-1,k-1}, \tilde{v}_{k-1}^{i-1}, \tilde{v}_{k-1}^{i}, \tilde{s}_{k-1}^{i-1}, \tilde{s}_{k-1}^{i}]^T \ ,$$

$$b_{\theta(k),\delta s_{i-1}} = [n_2, 0,1,0,0,0,0,0]^T,$$

$$b_{\theta(k),a_l} = [\emptyset_{0,k}, 0,1,0, Tj_k^{i-1}, T\, j_k^i, \tfrac{T^2}{2} j_k^{i-1}, \tfrac{T^2}{2} j_k^i]^T$$

$$c_{\theta(k)} = [1,0,0,0,0,0,0,0]$$

$$\theta_{1,k} = \tfrac{T^2}{2} k_2 j_k^{i-1} + \tfrac{T^2}{2} j {k_3}_k^{i-1} T + \tfrac{T^2}{2} k_2$$

$$\theta_{0,k} = -\tfrac{T^2}{2} k_2 j_k^i - \tfrac{T^2}{2} j {k_3}_k^{i} T - \tfrac{T^2}{2} k_2$$

$$\varphi_{1,k} = \tfrac{T^2}{2} k_3 j_k^{i-1} + \tfrac{T^2}{2} k_3$$

$\varphi_{0,k} = -\frac{T^2}{2}k_3 j_k^{i-1} - \frac{T^2}{2}k_3$

$\Psi_{1,k} = \frac{T^3}{2}k_2(j_k^i - j_k^{i-1}) + \frac{T^4}{2}k_3(j_k^i - j_k^{i-1}$

Stabilizing this system is not as simple as it was without delay. Now we have system we communication delay with two switching dynamics. Now we can't stabilize this system by just finding parameters satisfying condition$\|h(k)\|_1 \leq 1$. Now we have to find switching sequence that can minimize the error and a control law to make system string stable for any arbitrary switching sequence.

CHAPTER 3

STEEPEST DESCENT METHOD

3.1 Steepest Descent for Constrained Minimization

This method is given by Rodney Teo and Claire J Tomlin [9] and they have stabilized the system by finding maximum worst case error and optimizing it using steepest descent method. Steepest descent method constraint optimization has been used as we have some objective to achieve. Penalty function has been used for the converting constrained optimization to unconstrained optimization [17] [18].

3.1.1 Steepest Descent Method

Steepest descent is one of the oldest and simplest minimization technique it is also called as Cauchy’s method or gradient method.

$$\text{Minimize } f(x)$$

$$\text{Subject to } x \in R^n$$

Where $f: R^n \to R$ is convex and continuously differentiable.

Direction of minimization is opposite of the gradient $\nabla f(x_i)$ search starts at arbitrary point x_0. Iteration is given by

$$\boldsymbol{x_{k+1} = x_k - \lambda_k \nabla f(x_k) = x_k - \lambda_k g(x_k)} \qquad \textbf{3.1}$$

Where $g(x_k)$ is the gradient at one point.

Iteration is done until directional derivative is zero.

$$x_k - x_{k+1} < 0$$

When difference between iteration $i \; and \; i - 1$ is negative means it crossed the minimum point and now it’s again increasing.

3.1.2 Penalty Function

Penalty function is use to convert constraint minimization to unconstrained minimization. By applying penalty for the multiple constraints that can be violated function can be minimized [23]. If minimization of$f(x)$ subjected to

$$g_j(x) \leq 0, i = 1, \dots \dots \dots .. p$$

$$h_j(x) = 0, i = 1, \dots \dots \dots .. m$$

To minimize $f(x)$ we need to minimize $P(x)$.

Where,

$$\boldsymbol{P(x, \rho, \beta) = f(x) + \sum_{j=1}^{m} \rho \, h_j^2(x) + \sum_{j=1}^{m} \rho \, g_j^2(x)} \qquad \textbf{3.2}$$

The penalty parameters ρ_j and β_jb are given by

$$\rho_j >> 0$$

$$\beta_j = \begin{cases} 0 & if\ \beta_j(x) \leq 0 \\ \rho_j >> 0 & if\ \beta_j(x) \leq 0 \end{cases}$$

Square penalty function has been used in this case.

3.2 Worst Case

As in last section we can stabilize the system by minimizing the worst case condition that will ensure if system is stable in worst case it is stable in all other cases. We need to find values of parameter that gives us minimum worst case error.

From equation (1.9) maximum value of error $\delta s_{i,k}$ can be written as.

$$\boldsymbol{\delta s_{i,k}} = \begin{bmatrix} c_{\theta(k)}A_{\theta(k-1)}A_{\theta(k-2)}\ldots A_{\theta(1)}b_{\theta(0),a_{l,}} \\ c_{\theta(k)}A_{\theta(k-1)}A_{\theta(k-2)}\ldots A_{\theta(2)}b_{\theta(1),a_{l,}} \\ . \\ . \\ c_{\theta(k)}b_{\theta(k-1),a_{l,}} \end{bmatrix}^T \times \begin{bmatrix} \delta s_{i-1,0} \\ \delta s_{i-1,1} \\ . \\ . \\ \delta s_{i-1,k} \end{bmatrix} + \begin{bmatrix} c_{\theta(k)}A_{\theta(k-1)}A_{\theta(k-2)}\ldots A_{\theta(1)}b_{\theta(0),a_{l,}} \\ c_{\theta(k)}A_{\theta(k-1)}A_{\theta(k-2)}\ldots A_{\theta(2)}b_{\theta(1),a_{l,}} \\ . \\ . \\ c_{\theta(k)}b_{\theta(k-1),a_{l,}} \end{bmatrix}^T \times \begin{bmatrix} a_{l,0} \\ a_{l,1} \\ . \\ . \\ a_{l,k-1} \end{bmatrix} \qquad \mathbf{3.3}$$

We can separately study two components of $\delta s_{i,k}$ as

$$\boldsymbol{\delta s_{i,k} = \delta s_{i,k,\delta s_{i-1}} + \delta s_{i,k,a_l}} \qquad \mathbf{3.4}$$

$\|\delta s_{i,k}\|_\infty \leq \|\delta s_{i,k,\delta s_{i-1}}\|_\infty + \|\delta s_{i,k,a_l}\|_\infty$ Contribution of each input can be considered separately. δs_{i-1} Has no effect of communication dropouts. As δs_{i-1} has no effect of link status parameter j_k^i. Transfer function from δs_{i-1} to δs_i is same as it in condition when there is no dropout that means in case when link is always UP. Thus we can write the equation as

$$\boldsymbol{\|\delta s_{i,k}\| \leq \|h(k)\|_1 \|\delta s_{i-1}\|_\infty + \beta \|a_l\|_\infty} \qquad \mathbf{3.5}$$

Where β is∞ norm caused from a_l toδs_i.

Now we need to find the stability condition we need to consider contribution due to a_l which here is presented as β. And we can find which can be obtained by finding the switching and input sequences that will maximize the gain froma_l toδs_i. Now we will take very small sampling period as small sampling time is closest to the continuous time system. For sampling time T $d_1^2 = 4d_0, d_1 < 0 \; and \; d_0 > 0$ by which we can compute the value of β.

If$d_1^2 = 4d_0, d_1 < 0 \; and \; d_0 > 0$, $k_i > 0 \; for \; all \; i = 1,2,3, a, v, p$ and if ≤ 1, then the worst case maximum value of value of $\delta s_{i,k,a_l}$ is when $j_j^{i-1} = 0, j_j^i = 1, and \; a_{l,j} = 1$ or in other case when $j_j^{i-1} = 1, j_j^i = 0, and \; a_{l,j} = -1$ for $j = 0,1,2 \ldots \ldots k$.

On expansion of error due to a_l

$$\boldsymbol{\delta s_{i,k,a_l} = C_{\theta(k)} b_{\theta(k-1),a_l} a_{l,k-1} + C_{\theta(k)} A_{\theta(k-1)} b_{\theta(k-2),a_l} a_{l,k-2} \dots\dots\dots\dots\dots\dots\dots + C_{\theta(k)} A_{\theta(k-1)} A_{\theta(k-2)} \dots\dots A_{\theta(2)} b_{\theta(1),a_l} a_{l,1} + C_{\theta(k)} A_{\theta(k-1)} A_{\theta(k-2)} \dots\dots A_{\theta(1)} b_{\theta(0),a_l} a_{l,0}} \quad 3.6$$

It is understandable that the worst case condition happens when one of the vehicle's link or its preceding vehicle's received information is lossy. But when both have a lossy data reception it doesn't affect the system as it takes common information as reference. That means received information is not complete but it is for both vehicles. As we are using only one algorithm in both the vehicles it computes same results that's makes system stable when both link is down but not when both are down simultaneously. This condition is rare in practice and is not reliable but system stable whenever it occurs. This condition gives us a limit toβ.

We explained that if both vehicles losses data there is no instability in system but when only one vehicle receives loosed data and other has prefect data that means only one vehicle's link is down and others is UP gives us a worst case condition. It has big effects the spacing control of vehicle.

3.2.1 Stability Condition

As we can see in equation (3.6) that $\left\|\delta s_{i,k,a_l}\right\|_\infty$ becomes unbounded as no of k increases. If link is out for a large time system can't be stable as no k increases. In these cases we are taking bursty link that is UP for the maximum time and has link loss for a small time. That means DOWN time is very less. Of Consider instead burst links which will be UP for. When link is not available for the out timet_{out}, error$\delta s_{i,k,a_l}$could grow to stabilize the system. But it can be decreased when links comes back. Then $\delta s_{i,k,a_l}$will be attenuated over time. Let's assume that we have a minimum time gap between two outages oft_{up} . Assume that system has a settling time less thant_{up}. Then system gets stable before next outages and error will not be magnified as frequent outage occurs.

Let's assume $\alpha = \|h(k)\|_1$ which is gain of the vehicle without dropout, and consider effect from both input δs_{i-1} anda_l.

$$\|\boldsymbol{\delta s_i}\|_\propto \leq \boldsymbol{\alpha}\|\boldsymbol{\delta s_{i-1}}\|_\propto + \boldsymbol{\beta}\|\boldsymbol{a_l}\|_\propto \quad \mathbf{3.7}$$

$$\|\boldsymbol{\delta s_1}\|_\propto \leq \boldsymbol{\gamma}\|\boldsymbol{a_l}\|_\propto \quad \mathbf{3.8}$$

With $\beta > 0 \; and \; \gamma > 0$. This gives us worst case scenario of vehicles $1,2,3, \ldots \ldots i$, are cascaded together and will add up to give us a worst case accumulative effect on vehicle i as in equation (3.7) and (3.8)

$$\|\delta s_i\|_\propto \leq K\|a_l\|_\propto$$

Where,

$$\boldsymbol{K} = \frac{\boldsymbol{\alpha^i}}{\mathbf{1}-\boldsymbol{\alpha}}\left[\frac{\boldsymbol{\gamma}-\boldsymbol{\beta}}{\boldsymbol{\alpha}} - \boldsymbol{\gamma}\right] + \frac{\boldsymbol{\beta}}{\mathbf{1}-\boldsymbol{\alpha}} \quad \mathbf{3.9}$$

By gradient of K that will give us maxima of K by which we can find limits that can bound the error.

$$\frac{\boldsymbol{dK}}{\boldsymbol{di}} = \left[\frac{\boldsymbol{\gamma}(\mathbf{1}-\boldsymbol{\alpha})-\boldsymbol{\beta}}{(\mathbf{1}-\boldsymbol{\alpha})\boldsymbol{\alpha}}\right](\ln \boldsymbol{\alpha})\boldsymbol{\alpha^i} \quad \mathbf{3.10}$$

That is for all vehicles $i = 1,2, \ldots.$

We can get a conclusion from here that K will increase monotonically up to $\frac{\beta}{1-\alpha}$ if$\gamma \leq \beta/(1-\alpha)$, and K will monotonically decreases to $\frac{\beta}{1-\alpha}$ if$\gamma \leq \beta/(1-\alpha)$, as both are maxima and minima of K .

Then we write that

$$\gamma \leq \frac{\beta}{1-\alpha}$$

$$\leq \frac{\beta}{1-\alpha}\|a_l\|_\propto$$

It shows that error due to acceleration of lead vehicle β is bounded by this limit.

$$\gamma > \frac{\beta}{1-\alpha}$$

$$\leq \gamma\|a_l\|_\propto$$

2nd condition gives us the condition in which error attenuates which is string stable condition. In first case error is bounded that means it doesn't increases along platoon, that is weak sense of stability whereas 2nd condition gives us strong stability condition.

Spacing error will attenuate the maximum peak value by some factor of the lead vehicle's which makes all vehicle string stable.

We can conclude that if lead vehicle information is not available to us, by taking appropriate values of parameters and controller gains system will be string stable even in absence of the lead vehicle information. As error goes on decreasing as propagates down the stream.

Another conclusion we can draw from here is if we are taking settling time of the system t_s^* and we can calculate value of β for the time when link is outt_{out}. If lead vehicle has peak acceleration $\|a_l\|_\propto$ and there is a minimum time duration between two consecutive link DOWN oft_s^*. That guarantees system is string stable.

3.3 Numerical Analysis

Maximum absolute error obtained can be minimized using steepest descent method. Here we have some constraints to consider for minimization.

$$\boldsymbol{\beta}_{t_{out}} = \begin{bmatrix} c_{\theta(k)}A_{\theta(k-1)}A_{\theta(k-2)}\cdots A_{\theta(1)}b_{\theta(0),a_{l,}} \\ c_{\theta(k)}A_{\theta(k-1)}A_{\theta(k-2)}\cdots A_{\theta(2)}b_{\theta(1),a_{l,}} \\ . \\ . \\ c_{\theta(k)}b_{\theta(k-1),a_{l,}} \end{bmatrix}^{T} \times \begin{bmatrix} a_{l,0} \\ a_{l,1} \\ . \\ . \\ a_{l,k-1} \end{bmatrix} \qquad \textbf{3.11}$$

Minimization of $\beta_{t_{out}}$ gives the value of parameters. For $t_{up} = 3\ sec$ and $t_{out} = 1sec$ we can find β that on minimization can give parameter $g = [k_1\ , k_2,\ k_3,\ k_a,\ k_v\ , k_p]\ .k_i > 0\ for\ i = 1,2,3, a, v, p.$

Considering all minimizing condition we are getting the constraints

Minimization condition:-

1. ${}^{1.8}/_{\omega_n} \le t_r = 2.25$

2. ${}^{4.6}/_{\sigma} \leq 3$

3. ${}^{\sigma}/_{\omega_n} \geq \tau^* = 0.7$

4. $\|h(k)\|_1 \leq \alpha^* = 0.4$

5. $\|g\|_\infty \leq k_{max} = 2$

These conditions gives us the inequality constraints for the minimization.

1. $2k_a + .1k_v + .005k_p \geq 0.003$

2. $k_a + .1k_v + .005k_p \geq 0.0028$

3. $-2 + .1(k_v + k_2) + 0.005(k_p + k_3) \geq -1.77$

4. $1 - .1(k_v + k_2) + 0.005(k_p + k_3) \leq -0.78$

5. $k_1 + k_2 + k_3 + k_4 + k_5 + k_6 \leq 2$

But a constraint problem can't be minimized directly for that we need to first convert the constraint problem to unconstrained problem. Penalty function can be used for the purpose. A penalty function term is added to the minimizing function to set cost to the minimization.

$\tilde{f}(x) = f(x) + \sum_{i=1}^{m} P_i(x)$ i=1, 2…n

Where $f(x)$ is our function and $P_i(x)$ is square penalty function.Now using matlab and writing program for steepest descent and penalty function using

Parameters	values
k_2	4.328949e-01
k_3	4.376272e-01
k_a	2.177480e-01
k_v	4.783521e-01
k_p	4.403487e-01

Table 1 Steepest Descent Output Parameters

CHAPTER 4

A STATE-SPACE DESIGN APPROACH

4.1 Stability of Discrete Time Switched System Using LMI

From the previous section we can see that we have a discrete time switched system with two switching dynamics. System parameters have uncertainties that need to be found first.

Finding $[k_2,\ k_3,\ k_a,\ k_v\ ,k_p]$ for a stable $A_{\theta(k)}$ is a problem.

$$A_{\theta(k)} = \begin{bmatrix} -d_1 & -d_0 & n_1 & n_0 & \theta_{1,k} & \theta_{0,k} & \varphi_{1,k} & \varphi_{0,k} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j_k^{i-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j_k^i & 0 & 0 \\ 0 & 0 & 0 & 0 & Tj_k^{i-1} & 0 & j_k^{i-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & Tj_k^i & 0 & j_k^i \end{bmatrix}$$

We have unknowns$L = [k_2,\ k_3,\ k_a,\ k_v\ ,k_p]$.

Here we are trying SOF method to find the values of unknowns for stable dynamics in order to find these unknowns for a stable $A_{\theta(k)}$ we will be using static output feedback control.

4.2 Uncertain Dynamics using SOF

For SOF method first we need to convert $A_{\theta(k)}$ in to form to apply SOF.
We can convert $A_{\theta(k)}$ as follows

$$\boldsymbol{A_{\sigma(k)}} = (\boldsymbol{A_{\sigma(k)}} + \boldsymbol{B_{\sigma(k)} L C_{\sigma(k)}}) \qquad \textbf{4.1}$$

By applying static output feedback control L can be found and it will give us stable $A_{\theta(k)}$.

4.2.1 SOF and BMI Condition

Last section showed the condition to find unknown parameters. Here our unknown parameters doesn't change its value but it remains same in both condition that means a single controller gain should stabilize both switching instant dynamics.

$$\boldsymbol{x(k+1) = A_{\sigma(k)} x(k) + B^s_{\sigma(k)} u(k)} \qquad \textbf{4.2}$$

$$\boldsymbol{y(k) = C_{\sigma(k)} x(k)} \qquad \textbf{4.3}$$

Where,

$u(k) \in IR^m$ Is the control output vector, $y(k) \in IR^p$ is the measured output vector and $x(k) \in IR^n$ is the state vector.

Switching rule has finite set of values

$$\sigma(k) \in I = \{1,2,3 \dots \dots \dots . . q\}$$

And values of switching changes arbitrary. Because of the discrete time system it has different modes at discrete intervals. We have set of parameters for the switching modes.

$$A_i, B_i, C_i \quad i \in I$$

$\sigma(k)$Gives switching sequence for the system.

We are taking some assumption for the further synthesis.

- Switching sequence is not known priory but it is available at the instants.
- For every switching instant parameters A_i, B_i^s and A_i, C_i is controllable and observable.
- MatricesB_i^s has particular format to apply this theorem, that is

$$B_i^s = \begin{bmatrix} B_i \\ 0 \end{bmatrix} \qquad i \in I$$

Where$B_i \in IR^{m \times m}$ are square matrices.

For static output feedback control law would be

$$\boldsymbol{u(k) = L_{\sigma(k)} y(k)} \qquad \textbf{4.4}$$

Where $L_{\sigma(k)} \in \{L_i \in IR^{m \times p}\}$

Such that closed loop system becomes asymptotically stable.

$$\boldsymbol{x(k+1) = (A_{\sigma(k)} + B_{\sigma(k)} L_{\sigma(k)} C_{\sigma(k)}) x(k)} \qquad \textbf{4.5}$$

Now we need to define the Indicator function

$$\mu_i(k) = \begin{cases} 1 \; if \; system \; is \; in \; the \; i^{th} \; mode \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

Where $i = 1,2, \dots \dots \dots q$

We can write switched system matrix as follows

$$A_{\sigma(k)} = \sum_{i=1}^{q} \mu_i(k) A_i$$

$$B_{\sigma(k)} = \sum_{i=1}^{q} \mu_i(k) B_i^s$$

$$C_{\sigma(k)} = \sum_{i=1}^{q} \mu_i(k) C_i$$

Then closed loop state matrix becomes

$$\boldsymbol{A_{\sigma(k)} + B_{\sigma(k)} L_{\sigma(k)} C_{\sigma(k)} = \sum_{i=1}^{q} \mu_i(k)(A_i + B_i^s L_i C_i} \qquad \textbf{4.6}$$

Here for the static output feedback problem we are taking switched lyapunov function and congruence transformation is used to find a control law.

To guarantee asymptotic stability we are taking a particular type of lyapunov function. Switched quadratic lyapunov function that ensures switching of function as system has switching property. That means lyapunov function has q no of modes if system has q no of switching instants.

$$\boldsymbol{V\big(k, x(k)\big) = x^T(k) P_{\sigma(k)} x(k) = x^T(k)(\sum_{i=1}^{q} \mu_i(k) P_i) x(k)} \qquad \textbf{4.7}$$

With $P_i, i = 1,2,3 \ldots \ldots \ldots .. q$

If lyapunov function symmetric positive definite it will always guarantee the stability of system.

As we lyapunov stability theorem sates that, any system is quadratic asymptotically stable if and only if it satisfies the following condition.

$$\boldsymbol{A_{cl_i}^T P_j A_{cl_i} - P_i < 0 \qquad \forall (i, j) \in I \times I} \qquad \textbf{4.8}$$

Where $A_{cl_i} = A_{\sigma(k)} + B_{\sigma(k)} L C_{\sigma(k)}$

4.2.2 Transformation of BMI to LMI

Condition in last section $A_{cl_i} = A_{\sigma(k)} + B_{\sigma(k)} L C_{\sigma(k)}$ gives us BMI condition we have two unknowns L and P_j . Convex optimization of Bilinear Matrix Inequalities (BMI) condition is not simple for convex optimization. Two unknowns makes it difficult to solve. It is always better to solve LMI than BMI, it has quadratic term in which case it is difficult to find feasibility condition. We can covert BMI in LMI by using theory given in [18].

Any symmetric positive matrix likeP_i can be written as following matrix. It is partition of matrix in appropriate format.

$$\boldsymbol{P_i} = \begin{bmatrix} \boldsymbol{p_i^{11}} & \boldsymbol{p_i^{12}} \\ \boldsymbol{p_i^{12^T}} & \boldsymbol{p_i^{22}} \end{bmatrix} \qquad \textbf{4.9}$$

Where

$p_i^{11} \in IR^{m\times m}Andp_i^{11} \in IR^{n-m\times n-m}$.

Then $p_i^{11} > 0$ and $p_i^{22} - p_i^{12^T} p_i^{11^{-1}} p_i^{12} > 0$

Applying the schur complement and changing notations

$$p_i^1 = p_i^{11}$$

$$N_i = p_i^{11^{-1}} p_i^{12}$$

$$p_i^2 = p_i^{22} - p_i^{12^T} p_i^{11^{-1}} p_i^{12}$$

p_iCan be written as

$$\boldsymbol{p_i} = \begin{bmatrix} \boldsymbol{p_i^1} & \boldsymbol{p_i^1 N_i} \\ \boldsymbol{N_i^T p_i^1} & \boldsymbol{p_i^2 + N_i^T p_i^1 N_i} \end{bmatrix} = \boldsymbol{T_{N_i} P_{d_i} T_{N_i}^T} \qquad \textbf{4.10}$$

Where

$$P_{d_i} = diag(p_i^1, p_i^2) and\ T_{N_i} = \begin{bmatrix} I_m & 0 \\ N_i^T & I_{n-m} \end{bmatrix}$$

Where $T_{N_i}$is a type non-singular matrix. By congruence transformation we can say that the matrix p_i meets our condition only if it satisfies the above condition. $p_i > 0$If only if P_{d_i} for any matrixN_i.

We are investigating controller synthesis problem for the present system using static output feedback controller with switched quadratic lyapunov function.

The system will be stabilized by switched static output feedback controller if it satisfies the following conditions. If satisfies the following condition we get new LMI condition.

If there exist symmetric positive –definite matrices $W_i \in IR^{m\times m}$and$Q_i \in IR^{(n-m)\times(n-m)}$, general matrices $M_i \in IR^{m\times(n-m)}$, $R_i \in IR^{m\times p}$ and scalar $\delta > 0$ such that following inequalities holds for all

$$\begin{bmatrix} \boldsymbol{A_i^T \begin{bmatrix} 0 & 0 \\ 0 & Q_j \end{bmatrix} A_i - \begin{bmatrix} \delta I_m & M_i \\ M_i^T & Q_i \end{bmatrix}} & (*) \\ \boldsymbol{[\delta I_m \quad M_j] A_i + B_i R_i C_i} & \boldsymbol{-W_j} \end{bmatrix} \boldsymbol{< 0} \quad \textbf{4. 11}$$

$$\boldsymbol{W_j < \delta I_m} \quad \textbf{4. 12}$$

Holds for all$(i, j) \in I \times I$.

Then switched controller gain is given by

$$\boldsymbol{L_i = \delta^{-1} R_i} \quad \textbf{4. 13}$$

For all$i \in I$.

Proof of stability of LMI condition.

The closed loop system for switched lyapunov function is stable if and only if there exist $p_i > 0$ such that inequalities hold.

As we have seen in by schur complement that $p_i > 0$ is equal to $p_i^1 > 0$and$p_i^2 > 0$.

After using assumption in last equation we get

$$\boldsymbol{P_j B_i^s = \begin{bmatrix} p_j^{11} \\ p_j^{12^T} \end{bmatrix} B_i = \begin{bmatrix} I_m \\ N_j^T \end{bmatrix} p_j^{(1)} B_i} \quad \textbf{4. 14}$$

Then the inequalities are equivalent to

$$\boldsymbol{A_i^T P_j A_i - P_i - A_i^T \begin{bmatrix} I_m \\ N_j^T \end{bmatrix} p_j^{(1)} B_i L_i C_i + C_i^T L_i^T B_i^T P_j^1 [I_m \quad N_j] A_i + (B_i L_i C_i)^T P_j^1 (B_i L_i C_i) < 0}$$

4. 15

Which in turn are equivalent to

$$\boldsymbol{A_i^T P_j A_i - P_i - A_i^T \begin{bmatrix} I_m \\ N_j^T \end{bmatrix} p_j^{(1)} [I_m \quad N_j] A_i + \tau_{ij}^T P_j^1 \tau_{ij}^T < 0 \quad \forall (i,j) \in I \times I} \qquad \textbf{4. 16}$$

Where

$$\tau_{ij} = [I_m \quad N_j] A_i + B_i L_i C_i$$

As matrices p_i^1 are positive definite, there exist a scalar $\delta > 0$ such that $p_i^1 > \delta I_m$ for all

$i \in I$.

$$\boldsymbol{P_i = \begin{bmatrix} 0 & 0 \\ 0 & p_i^{(2)} \end{bmatrix} + \begin{bmatrix} I_m \\ N_j^T \end{bmatrix} p_i^{(1)} [I_m \quad N_j] \geq \begin{bmatrix} \delta I_m & \delta N_i \\ \delta N_i^T & p_i^{(2)} \end{bmatrix}} \qquad \textbf{4. 17}$$

Using inequality and fact that

We can say that are satisfied if the following inequalities hold

$$\boldsymbol{A_i^T P_j A_i - A_i^T \begin{bmatrix} I_m \\ N_j^T \end{bmatrix} p_j^{(1)} [I_m \quad N_j] A_i = A_i^T \begin{bmatrix} 0 & 0 \\ 0 & p_j^{(2)} \end{bmatrix} A_i} \qquad \textbf{4. 18}$$

$$\boldsymbol{A_i^T \begin{bmatrix} 0 & 0 \\ 0 & p_j^{(2)} \end{bmatrix} A_i - \begin{bmatrix} \delta I_m & \delta N_i \\ \delta N_i^T & p_i^{(2)} \end{bmatrix} + \tau_{ij}^T P_j^1 \tau_{ij}^T < 0 \quad \forall (i,j) \in I \times I} \qquad \textbf{4. 19}$$

By schur complement, are equivalent to

$$\boldsymbol{\begin{bmatrix} A_i^T \begin{bmatrix} 0 & 0 \\ 0 & p_j^{(2)} \end{bmatrix} A_i - \begin{bmatrix} \delta I_m & \delta N_i \\ \delta N_i^T & p_i^{(2)} \end{bmatrix} & (*) \\ [I_m \quad N_j] A_i + B_i L_i C_i & -p_j^{1^{-1}} \end{bmatrix} < 0} \qquad \textbf{4. 20}$$

After multiplication with the following to right and transpose to left

$$\begin{bmatrix} I_n & 0 \\ 0 & \delta I_m \end{bmatrix}$$

We get the following inequalities

$$\begin{bmatrix} \boldsymbol{A_i^T \begin{bmatrix} 0 & 0 \\ 0 & p_j^{(2)} \end{bmatrix} A_i - \begin{bmatrix} \delta I_m & \delta N_i \\ \delta N_i^T & p_i^{(2)} \end{bmatrix}} & (*) \\ \boldsymbol{[\delta I_m \quad \delta N_j] A_i + \delta B_i L_i C_i} & \boldsymbol{-\delta^2 p_j^{1^{-1}}} \end{bmatrix} < \boldsymbol{0} \qquad \textbf{4.21}$$

4.2.3 LMI Condition

LMI condition obtained in last section can be used to find the unknown parameters.

$$\begin{bmatrix} A_i^T \begin{bmatrix} 0 & 0 \\ 0 & p_j^{(2)} \end{bmatrix} A_i - \begin{bmatrix} \delta I_m & \delta N_i \\ \delta N_i^T & p_i^{(2)} \end{bmatrix} & (*) \\ [\delta I_m \quad \delta N_j] A_i + \delta B_i L_i C_i & -\delta^2 p_j^{1^{-1}} \end{bmatrix} < 0$$

Now we need to change some of the notations to simplify our equation in order to have a simple calculation

$$M_i = \delta N_i, R_i = \delta L_i, Q_i = p_i^{(2)}, W_j = \delta^2 p_j^{1^{-1}}$$

On replacing notations in the above inequality we get the condition. Because order to have $W_j \ and \ Q_i$ positive-definite,we must have p_i^1 and p_i^2 positive-definite. We can say that following condition must satisfy. $p_i^1 > \delta I_m$ Which is equivalent to the condition$\delta^2 p_j^{1^{-1}} > \delta I_m$.

Now using notation that we defined we will get a new LMI condition (5) by combining with other conditions we get the new condition that is suitable for the convex optimization technique.

Where variables in LMI areM_i , Q_i, R_i, W_iandδ. That will give us the output feedback gain.

$$L_i = \delta^{-1} R_i$$

We have assumed lyapunov function that will guarantee the stability of closed loop system. We can calculate the lyapunov function as follows

$p_i^1 = \delta^2 W_i^{1^{-1}}$,$p_i^2 = Q_i$, $N_i = \delta^{-1} M_i$ When M_i , Q_i, R_i, W_iandδ.

Consider a partitioning of the matrices as the partitioning of the matrix P_i

$$A = \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix}$$

Where $A_i^{11} \in IR^{m \times m}$

If $A_i^{11}$of matrix is negative definite system is stable, it is a necessary condition. Matrix A_i^{22} should be schur stable.After applying this LMI condition to find the unknown parameter of the system we get a stable system matrix.

4.3 Controller Design

After finding unknown parameters we have complete dynamics of the system. Now big problem is to find a controller to stabilize the system. As we have discrete time switched linear system we can use theory given in [19]. This uses switched lyapunov approach to find controller using LMI with convex optimization. From previous LMI condition we have unknown parameters of $A_{\sigma(k)}$ which is stable$L = [k_2,\ k_3,\ k_a,\ k_v\ ,k_p]$. Our complete discrete time switched linear system is

$$\boldsymbol{A_{\sigma(k)}} = \begin{bmatrix} -d_1 & -d_0 & n_1 & n_0 & \theta_{1,k} & \theta_{0,k} & \varphi_{1,k} & \varphi_{0,k} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j_k^{i-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j_k^{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & Tj_k^{i-1} & 0 & j_k^{i-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & Tj_k^{i} & 0 & j_k^{i} \end{bmatrix} \tag{4.22}$$

$$\boldsymbol{x_{k+1} = A_{\theta(k)}x(k) + [b_{\theta(k),\delta s_{i-1}}, b_{\theta(k),a_{l,}}]u_k} \tag{4.23}$$

$$\boldsymbol{y(k) = c_{\theta(k)}x(k)} \tag{4.24}$$

Now we have our system with stable$A_{\theta\sigma(k)}$.

This can also be written as format to process.

$$\boldsymbol{x(k+1) = A_{\sigma(k)}x(k) + B^s_{\sigma(k)}u(k)} \tag{4.25}$$

$$\boldsymbol{y(k) = C_{\sigma(k)}x(k)} \tag{4.26}$$

Where $x(k) \in IR^n$ is the state vector, $u(k) \in IR^m$ is the control output vector and $y(k) \in IR^p$ is the measured output vector.

Here we are taking some assumption

1. Switching rule is not known priori but it is at real time.
2. Matrix C_α is full row rank.

4.3.1 Switched Lyapunov Theorem for Discrete Time Switched System.

For Static output feedback controller we have control input. Here also we don't know the switching condition and don't know when to apply what controller gain, so we need only one controller gain that can stabilize system's both dynamics. That will also give us controller that will stabilize the system irrespective of the switching that is a big advantage .

$$u_k = Ky_k$$

To guarantee closed loop stability

$$\boldsymbol{x(k+1) = (A_\alpha + B_\alpha K C_\alpha) x(k)} \qquad \textbf{4.27}$$

Defining indicator function

$$\mu_i(k) = \begin{cases} 1 \; if \; system \; is \; in \; the \; i^{th} \; mode \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

Lyapunov function is

$$V\big(k, x(k)\big) = x^T(k) P_\alpha x(k)$$

For a switched system switched Lyapunov function can be written as

$$\boldsymbol{V\big(k, x(k)\big) = x^T(k) \left(\textstyle\sum_{i=1}^{N} \mu_i(k)\, P_i\right) x(k)} \qquad \textbf{4.28}$$

Any system is stable with Lyapunov function if and only if

1. If Lyapunov function V is positive definite function.
2. $\Delta V\big(k, x(k)\big)$Is negative definite.

Theorem: -

1) If function satisfies the above condition system would be asymptotically stable.

2) Then there is N symmetric matrices, that can satisfy the following condition

$$\begin{bmatrix} P_i & A_i^T P_j \\ P_j A_i & P_j \end{bmatrix} > 0 \qquad \forall (i,j) \in I \times I$$

Then we get our Lyapunov function

$$\boldsymbol{V\big(k, x(k)\big) = x^T(k)(\textstyle\sum_{i=1}^{N} \mu_i(k)\, P_i)x(k)} \qquad \textbf{4.29}$$

3) Then there is N symmetric matrices, that can satisfy the following condition

$$\begin{bmatrix} \boldsymbol{G_i + G_i^T - S_i} & \boldsymbol{G_i A_i^T} \\ \boldsymbol{A_i G_i} & \boldsymbol{S_j - A_i S_i A_i^T} \end{bmatrix} > \boldsymbol{0} \qquad \forall (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \qquad \textbf{4.30}$$

Then we get Lyapunov function as

$$\boldsymbol{V\big(k, x(k)\big) = x^T(k)(\textstyle\sum_{i=1}^{N} \mu_i(k)\, S_i^{-1})x(k)} \qquad \textbf{4.31}$$

Proof: - Condition 3 can be

$$\boldsymbol{G_i + G_i^T - S_i > 0} \quad \forall \ \boldsymbol{i} \in \boldsymbol{I} \qquad \textbf{4.32}$$

That says matrices G_i is of full rank and S_i is positive definite. We can write the condition as follows

$$\boldsymbol{(S_i - G_i)^T S_i^{-1} (S_i - G_i) \geq 0} \quad \forall \ \boldsymbol{i} \in \boldsymbol{I} \qquad \textbf{4.33}$$

That can also be written as

$$\boldsymbol{G_i^T S_i^{-1} G_i \geq G_i^T + G_i - S_i} \forall \ \boldsymbol{i} \in \boldsymbol{I} \qquad \textbf{4.34}$$

If condition holds, we can take

$$\begin{bmatrix} \boldsymbol{G_i^T S_i^{-1} G_i} & \boldsymbol{G_i^T A_i^T} \\ \boldsymbol{A_i G_i} & \boldsymbol{S_j} \end{bmatrix} < \boldsymbol{0} \qquad \forall (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \qquad \textbf{4.35}$$

That can also be written as

$$\begin{bmatrix} \boldsymbol{G_i^T} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S_j} \end{bmatrix} \begin{bmatrix} \boldsymbol{S_i^{-1}} & \boldsymbol{A_i^T S_j^{-1}} \\ \boldsymbol{S_j^{-1} A_i} & \boldsymbol{S_j^{-1}} \end{bmatrix} \begin{bmatrix} \boldsymbol{G_i} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S_j} \end{bmatrix} > \boldsymbol{0} \qquad \forall (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \qquad \textbf{4.36}$$

Assuming $P_i = S_i^{-1} P_j = S_j^{-1}$ then is equivalent to

$$\begin{bmatrix} \boldsymbol{P_i} & \boldsymbol{A_i^T P_j} \\ \boldsymbol{P_j A_i} & \boldsymbol{P_j} \end{bmatrix} > \boldsymbol{0} \quad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \tag{4.37}$$

To prove theorem 2. By schur complement we can write

$$\boldsymbol{P_i - A_i^T P_j A_j > 0} \quad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \tag{4.38}$$

Let's assume $S_i = P_i^{-1} S_j = P_j^{-1}$after using schur complement we can write

$$\boldsymbol{S_i - A_i^T S_j A_j = T_{ij} > 0} \quad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \tag{4.39}$$

Assume $G_i = S_i + g_i I$ with g_i positive scalar, we can also write the equation as follows

$$\boldsymbol{g_1^{-2}(S_i + 2g_i I) > A_i^T T_{ij} A_i} \quad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \tag{4.40}$$

That is equivalent to the

$$\begin{bmatrix} \boldsymbol{S_i + 2g_i I} & \boldsymbol{-g_i A_i^T} \\ \boldsymbol{-A_i g_i} & \boldsymbol{T_{ij}} \end{bmatrix} > \boldsymbol{0} \quad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \tag{4.41}$$

Which is nothing more than

$$\begin{bmatrix} \boldsymbol{G_i + G_i^T - S_i} & \boldsymbol{S_i A_i^T - G_i A_i^T} \\ \boldsymbol{A_i S_i - A_i G_i} & \boldsymbol{S_j - A_i S_i A_i^T} \end{bmatrix} > \boldsymbol{0} \quad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{I} \times \boldsymbol{I} \tag{4.42}$$

We have LMI condition that can stabilize the system

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{-A_i} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{G_i + G_i^T - S_i} & \boldsymbol{G_i A_i^T} \\ \boldsymbol{A_i G_i} & \boldsymbol{S_j - A_i S_i A_i^T} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{-A_i^T} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} > \boldsymbol{0} \tag{4.43}$$

4.3.2 LMI Condition for Stability

From last section we got LMI condition for the system. We can use convex optimization to find the stable controller gain. Final LMI condition

If there exists symmetric matricesS_i, matricesG_i,V_i,U_i such that $\forall(i, j) \in I \times I$

$$\begin{bmatrix} \boldsymbol{G_i + G_i^T - S_i} & \boldsymbol{(A_i G_i + B_i U_i C_i))^T} \\ \boldsymbol{(A_i G_i + B_i U_i C_i)} & \boldsymbol{S_j} \end{bmatrix} > \boldsymbol{0} \tag{4.44}$$

And

$$\boldsymbol{V_i C_i = C_i G_i} \forall \ \boldsymbol{i \in I} \tag{4.45}$$

Then, output feedback control can be given as which can stabilize the system.

$$\boldsymbol{K_i = U V_i^{-1}} \forall \ \boldsymbol{i \in I} \tag{4.46}$$

CHAPTER 5

NUMERICAL ANALYSIS AND SIMULATION RESULTS

5.1 Uncertainty Modelling

From the last chapter we can see that we have a discrete time switched system with two switching dynamics. System parameters have uncertainties that need to be found first.

Finding $[k_2,\ k_3,\ k_a,\ k_v\ , k_p]$ for a stable $A_{\theta(k)}$ is a problem.

$$A_{\theta(k)} = \begin{bmatrix} -d_1 & -d_0 & n_1 & n_0 & \theta_{1,k} & \theta_{0,k} & \varphi_{1,k} & \varphi_{0,k} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j_k^{i-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j_k^{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & Tj_k^{i-1} & 0 & j_k^{i-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & Tj_k^{i} & 0 & j_k^{i} \end{bmatrix}.$$

First we need to convert $A_{\theta(k)}$in the form of static output feedback control that is

$$A_{\theta(k)} = A_{\sigma(k)} + B_{\sigma(k)} L C_{\sigma(k)}$$

Where L is $[k_2,\ k_3,\ k_a,\ k_v\ , k_p]$

$$A_{\theta(k)} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} A^{11} & A^{12} & A^{13} & A^{14} & A^{15} & A^{16} & A^{17} & A^{18} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where

$A^{11} = -(k_v + k_2)T + \frac{1}{2}T^2(k_p + k_3),$

$A^{12} = (k_v + k_2)T - \frac{1}{2}T^2\left(k_p + k_3\right),$

$A^{13} = -2k_a - k_v T + \frac{1}{2}k_p T^2,$

$A^{14} = k_a - k_v T + \frac{1}{2}k_p T^2,$

$A^{15} = \frac{1}{2}T^2 k_2,$

$A^{16} = T^2 k_2 - \frac{1}{2} T^2 k_3,$

$A^{17} = \frac{1}{2} T^2 k_3,$

$A^{18} = T^2 k_3.$

This can also be written as

$$A_{\theta(k)} = A_{\sigma(k)} + B_{\sigma(k)} K_{\sigma(k)} C_{\sigma(k)}$$

Where $\sigma(k) = [1,2\ ]$ here we have 2 switching instants.

$$A_1 = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .1 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -.1 & .1 & 0 & 0 & .005 & .01 & 0 & 0 \\ -.005 & -.005 & 0 & 0 & 0 & -.005 & .005 & -.01 \\ -.1 & .1 & .1 & -.1 & 0 & 0 & 0 & 0 \\ -.005 & -.005 & .005 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -.1 & .1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -.005 & -.005 & 0 & 0 & 0 & 0 & 0 & 0 \\ -.1 & .1 & .1 & -.1 & 0 & 0 & 0 & 0 \\ -.005 & -.005 & .005 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$B_i^s = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and

$L = [k_1\ , k_2,\ k_3,\ k_4,\ k_5].$

By writing LMI program using LMI condition given in [21] and applying that theory in this problem to find the one unknown parameter that can stabilize give stable $A_{\theta(k)}$ in both condition. Theory given in [21] is for switched controller gain which has been modified to get a single controller gain. From the LMI program and finding the feasibility condition by convex optimization we got the controller gain.

$L = [.00616, 0, 0.87364, 0.00248, 0.0047948].$

Uncertain Parameters	Output Values
k_2	$.00616$
k_3	0
k_a	$0.87364,$
k_v	0.00248
k_p	0.0047948

Table 2 Uncertain Parameters

On applying controller gain values found by program we have stable $A_{\theta(k)}$ for further analysis.

$$A_1 = \begin{bmatrix} 1.912 & -.912 & .0778 & -0.0825 & 3.08\times 10^{-5} & 6.16\times 10^{-5} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .1 & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.912 & -.912 & .0778 & -0.0825 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} .0048 & 3.08 \times 10^{-5} \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.1 \\ 0 & 0 \\ 0 & -0.005 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} .0048 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = [1, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0],$$

5.2 Controller Design

Now as we have system with all stable matrices we need to find a control law to stabilize the system for any arbitrary condition. Here also we are applying static output feedback control but due to condition like the assumption we took in last SOF problem we can’t apply that here. Stabilization of Switched discrete time linear system given in [22] we can find a controller.

We take control law as

$$u_k = K_\alpha y_k$$

To guarantee closed loop stability of

$x(k+1) = (A_\alpha + B_\alpha K C_\alpha)x(k),$

we have LMI condition for particular case from equation no (4.44) and (4.45)

Using LMI condition $\begin{bmatrix} G_i + G_i^T - S_i & (A_i G_i + B_i U_i C_i))^T \\ (A_i G_i + B_i U_i C_i) & S_j \end{bmatrix} > 0$

And

$$V_i C_i = C_i G_i \forall \ i \in I$$

Then, output feedback control can be given as which can stabilize the system.

$$K = U V_i^{-1} \forall \ i \in I$$

Using LMI programming and applying the control law we got the controller gain

$$K = \begin{bmatrix} -0.1647 \\ 0.8171 \end{bmatrix}$$

On applying this controller on the system have a stable system that has no effect of the switching.

$$A_{\theta(k)} = A_{\sigma(k)} + B_{\sigma(k)} K C_{\sigma(k)}$$

where,

$$A_1 = \begin{bmatrix} 1.9112 & -.9120 & .0778 & -0.0825 & 0.0001 & 0.001 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1647 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0817 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0041 & 0 & 0 & 0 & 0 & .1 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1.9112 & -.9120 & .0778 & -0.0825 & 0.0001 & 0.001 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1647 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0817 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0041 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} .0048 & 3.08 \times 10^{-5} \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.1 \\ 0 & 0 \\ 0 & -0.005 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} .0048 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = [1, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0],$$

5.3 Simulation Results

System dynamics given in last section is for a stable system. But we need to check the performance of system by designing a model in Simulink for a platoon of 4 vehicles following the lead vehicle.

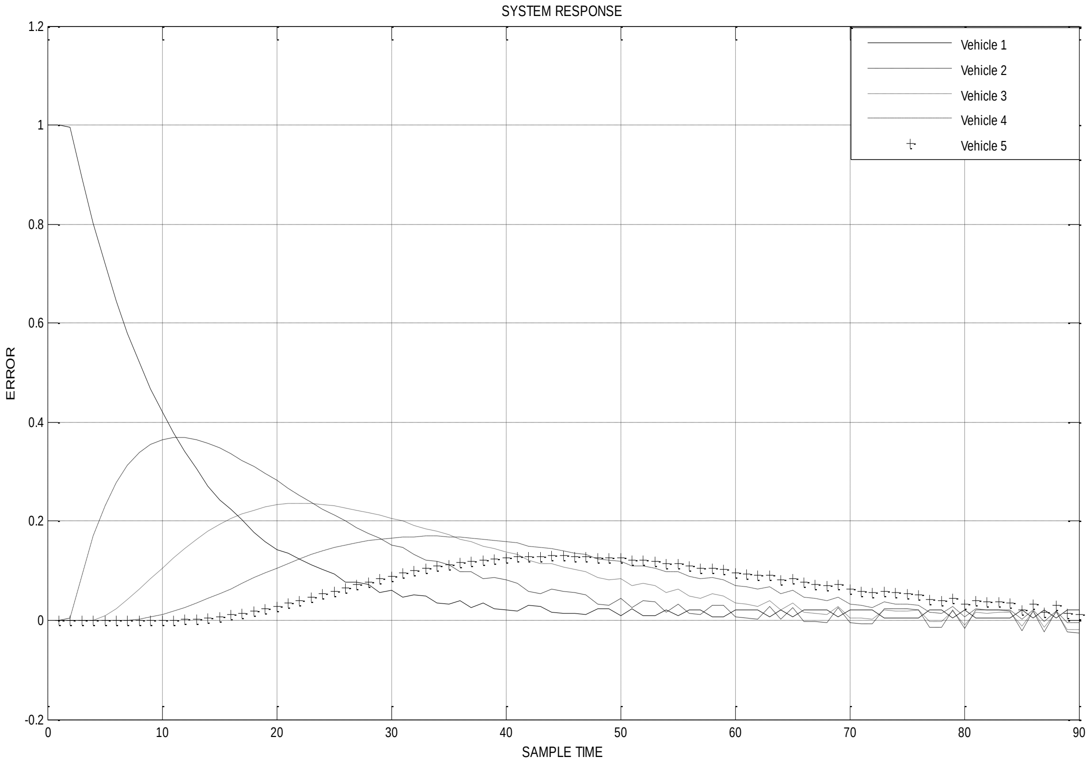

Figure 2 System Response

From simulation result we can see that peak spacing error of vehicle 1 is greater than that of vehicle 2. Peak spacing error is decaying as we go down the platoon for vehicle 3,4 and 5.

We can say that system is string stable for the random switching.

CHAPTER 5

CONCLUSION AND FUTUTRE SCOPE

6.1 Conclusion

In this thesis spacing has been maintained for a platoon of 5 vehicles at the time of communication delay, where all vehicles receive information of lead and preceding vehicles. It has been assumed that preceding vehicle information is prefect and only lead vehicle information suffers from communication delay. Switched Static Output feedback with LMI has been used for the modelling of uncertain system. Switched Static Output Feedback Controller has been designed to maintain string stability in case of communication delay. We can say from the simulation result, initial error in the peak spacing error of vehicles are decreasing as we are going down the platoon. Peak spacing error of second vehicle is less than that of first vehicle. State space modelling based design has advantage over earlier design as it has stabilized the system with arbitrary switching whereas steepest descent method has assumption in the pattern of switching and it's unstable with arbitrary switching.

6.2 Future Scope

1. In this thesis we have taken communication of preceding vehicle prefect in future communication delay in preceding vehicle information can also be considered.
2. Switched Static output Feedback Controller can be designed for the platoon with varying velocity of lead vehicle at the time of communication delay.

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